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REVIEW OF EXPERIMENTAL RESULTS ON PRECISION TESTS OF ELECTROWEAK THEORIES

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Abstract

The current status of precision electroweak measurements, including data from LEP, the Fermilab Tevatron and the SLC, is reviewed. The data include measurements of the masses and widths of the W and Z bosons, as well as other data on Z-fermion couplings from measurements of cross sections, forward-backward asymmetries, τ polarisation and the left-right asymmetry A_{LR}. The data are used to test lepton universality. The results from LEP and the SLC on heavy quark couplings have also reached interesting levels of precision. The current, and still preliminary, data give world average values of the ratios of the quark partial to hadronic widths for b and c quarks, R_{b} and R_{c}, which are about 3.7 and 2.5 standard deviations respectively from Standard Model (SM) fits to all electroweak data. Using these and other LEP and SLC data, the b and c quark-couplings are extracted. The global electroweak fits to all data give an indirect value of the top quark mass which is in good agreement with the direct measurement at the Tevatron by the CDF and D0 Collaborations; a significant triumph for the Standard Model. The influence of R_{b} and R_{c} on these fits is discussed. The global SM fits favour a light Higgs mass. However, the appropriate error scale for m_{H} is logarithmic and the upper limits are sensitive to the inclusion of the data on R_{b}, R_{c} and A_{LR}, all of which are somewhat discrepant from the SM expectations.

1. Introduction

The progress in precision tests of electroweak theories during the last few years has been remarkable. At the 1987 Lepton-Photon Symposium [1], which was before the impact of results from LEP, SLC and the Fermilab Tevatron, the errors on the Z and W boson masses (from the UA1 and UA2 experiments) were $\pm$ 1.8 GeV and $\pm$ 1.3 GeV respectively, compared to the current values of $\pm$ 2.2 MeV and $\pm$ 160 MeV. Since the 1993 Lepton-Photon Symposium [2] the major advance has been the discovery of the top-quark at the Tevatron, by the CDF [3] and D0 Collaborations [4], with a combined mass value of $m_{\rm t}$ = 180 $\pm$ 12 GeV. The agreement between this direct measurement and the indirect measurement from the effects of electroweak loop corrections is a remarkable triumph for the minimal Standard Model (SM). This comparison depends also on the Higgs Boson mass $m_{\rm H}$, and the current data start to give meaningful constraints also on $m_{\rm H}$. This agreement between the data and the SM also gives stringent limits on possible new physics beyond the SM. At present, the Z branching ratios to c and b-quarks and the left-right asymmetry measured at the SLC show the largest deviations and are discussed in more detail below.

The SM is specified by the $SU(3)_{col}\otimes SU(2)_L\otimes U(1)_Y$ coupling constants α_s, g and g$'$, together with the fermion parameters $\{m_f\}$ and those of the Higgs boson. In practice, the predictions are made in terms of the experimentally well measured parameters G_μ;[a] $m_{\rm Z}$ and $\alpha(m_{\rm Z})$, the value of the QED coupling at $\sqrt{s} = m_{\rm Z}$; together with $\alpha_{\rm s}(m_{\rm Z})$ and the parameters $m_{\rm t}$ and $m_{\rm H}$. Of course, genuine tests of the SM itself will become more meaningful when the Higgs mass and all parameters are precisely known.

2. The running of α

The fine structure constant is known at $q^2 \simeq 0$ with impressive precision (4 10^{-8}); however, what is important for the interpretation of heavy gauge boson results is $\alpha(m_{\rm Z})$. Here the precision is $\simeq$ 8 10^{-4}. The running of α is given by

$$\alpha(m_{\rm Z}) = \frac{\alpha(0)}{1-\Delta\alpha(m_{\rm Z})}, \tag{1}$$

where $\Delta\alpha(m_{\rm Z}) = \Delta\alpha_{lept} + \Delta\alpha_{had} = -\Pi_{\gamma\gamma}(s)$, with $\Pi_{\gamma\gamma}$ the photon self-energy. The leptonic part $\Delta\alpha_{lept}$ can be calculated analytically, and is well known. The hadronic

[a] G_μ is determined directly from the muon lifetime τ_μ, which is known to a precision of 0.9 10^{-5}. However, G_μ is determined only to a precision of 1.7 10^{-5}, the dominant error being from second order QED corrections, (Particle Data Group, Phys. ReV. D50 (1994))

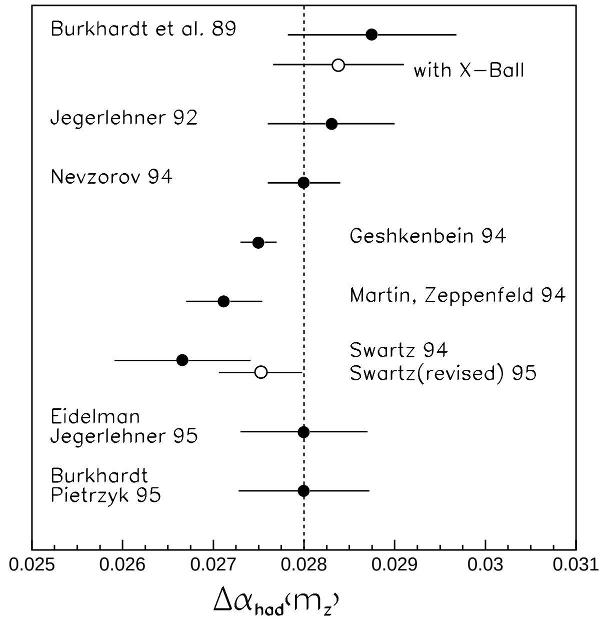

Fig. 1. Determinations of $\Delta\alpha_{had}(m_Z)$.

part $\Delta\alpha_{had}$ cannot be calculated entirely from QCD because of ambiguities in defining the light quark masses m_u and m_d and also the inherent non-perturbative nature of the problem at small energy scales.

Instead, use is made of the data on

$$R_{had}(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \tag{2}$$

from which one can compute

$$\mathrm{Re}\Pi_{\gamma\gamma}(s) = \frac{\alpha s}{3\pi} P \int \frac{R_{had}(s')}{s'(s'-s)} ds' \quad . \tag{3}$$

Most of the sensitivity is to R_{had} at low values of s. In practice, there are difficulties in evaluating the integral :-

a) from the resonant structure in the data (ρ,ω,J/ψ, Υ)

b) some data are not very accurate (eg. large systematic error) and old (not always enough information is given)

c) somewhat arbitrary choices need to be made in the use of the data (eg. the form of local parameterisation, interpolation or fit, how to deal with inconsistent data and how to cross thresholds etc.)

Fig. 1 shows a summary of the determinations of $\Delta\alpha_{had}$. In reference [5] it was shown that compatible results could be obtained from different methods. The results of references [6 7 8] rely significantly more on QCD than other estimates. The value from reference [9], $\Delta\alpha_{had} = 0.02752 \pm 0.00046$, which is obtained from fitting the data, is a revised value. The original value was considerably lower than those of the two most recent evaluations [10 11], which also rely rather directly on the data. The change is due mainly to a correction of a bias in the fit procedure and to the introduction of Crystal Ball data, just below the charm threshold.

In comparing the results in fig. 1 it should be borne in mind that the data used for these evaluations are largely the same. The determinations which rely heavily on QCD need to be treated with some caution as they are not entirely independent of the data, since QCD did not predict all the observed structure! It is proposed to use the value of the two most recent evaluations [10 11], namely

$$\alpha^{-1}(m_{\mathrm{Z}}) = 128.89 \pm 0.09. \tag{4}$$

Although the largest component of the integral (~ 45 %) is from the "continuum" region $\sqrt{s} > 12$ GeV, by far the largest uncertainty comes from the low s region. More than 75% of the total error is from the region $1 \leq \sqrt{s} \leq 5$ GeV. As discussed in section 6, the error on $\alpha(m_{\mathrm{Z}})$ is significant in the interpretation of electroweak fits. It is important that this error is reduced by making further precise measurements of R_{had} in this low energy range. Measurements at DaΦne, Novosibirsk and Beijing are thus strongly encouraged.

3. Measurement of M_{W} and Γ_{W}

The W boson mass M_{W} and width Γ_{W} have so far only been measured at hadron colliders with the latest, and most precise, results coming from the Fermilab Tevatron $\mathrm{p\bar{p}}$ collider at $\sqrt{s} = 1.8$ TeV. The data used come from 1992/3 (Run 1a).

The CDF Collaboration have published final results [12] on both the $\mathrm{W} \to \mathrm{e}\nu$ and $\mathrm{W} \to \mu\nu$ channels, based on 19 pb^{-1}. The analysis uses variables defined in the plane perpendicular to the incident beams. A high $\mathbf{p}_T^l$ lepton ($\mathrm{E}_T^l > 25$ GeV) is required. In addition, the transverse momentum $\mathbf{u}$ of the recoil against the W must be well determined, with $|\mathbf{u}| < 20$ GeV. The calibration of $\mathbf{u}$ is made on $\mathrm{Z} \to \mathrm{e^+e^-}$ decays. The missing neutrino vector $\mathbf{p}_T^\nu = -\ (\ \mathbf{p}_T^l + \mathbf{u}\ )$ can thus be reconstructed and $\mathrm{E}_T^\nu >$ 25 GeV is required. M_{W} is determined from a fit to the W transverse mass M_T^W, where

$$(M_T^W)^2 = 2\mathrm{p}_{\mathrm{T}}^\ell \mathrm{p}_{\mathrm{T}}^\nu (1 - cos\phi_{\ell\nu})\ . \tag{5}$$

In addition, the backgrounds from $\mathrm{W} \to \tau\nu$, $\mathrm{Z} \to \ell\ell$ etc must be minimised and care taken in the event selection that no bias is introduced.

The momentum scale is calibrated from J/ψ decays and cross-checked on Υ and Z decays, which are also used to determine the resolution. The energy scale for electrons is determined from detailed study of the electron sample itself, plus use of the momentum scale.

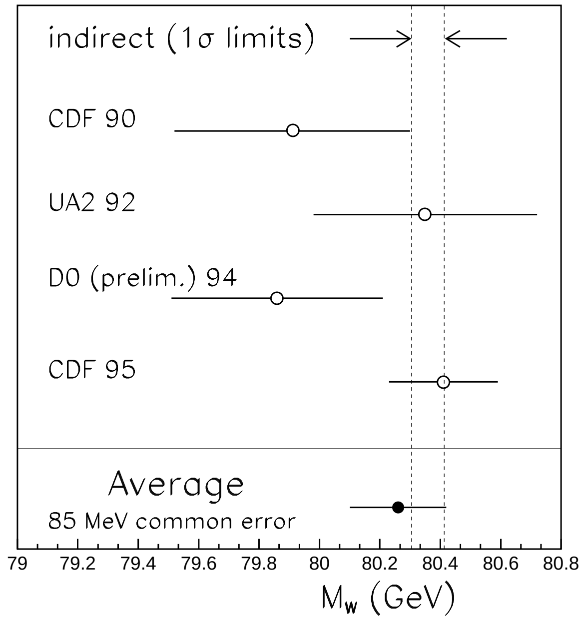

Fig. 2. Direct measurements of $M_{\rm W}$ with their average. The dashed lines are from the SM fit B.

The results from the 5718 and 3268 events in the selected $\rm W \to e\nu$ and $\rm W \to \mu\nu$ samples are $M_{\rm W}(\rm e\nu)$ = 80.490 ± 0.145 (stat) ± 0.175 (syst) GeV and $M_{\rm W}(\mu\nu)$ = 80.310 ± 0.205 (stat) ± 0.130 (syst) GeV. Combining these, taking into account the common systematic error (0.100 GeV), gives

$$M_{\rm W} = 80.410 \pm 0.180 \quad \text{GeV}. \tag{6}$$

The D0 Collaboration have a preliminary result using the $\rm W \to e\nu$ channel, based on $\simeq$ 13 $\rm pb^{-1}$. The energy scale is determined from $\rm Z \to e^+e^-$ decays, giving a contribution to the systematic error of $\delta {\rm M}_W^{syst}$ = ± 260 MeV. The D0 result, which is that reported at Glasgow in 1994 [13], is

$$M_{\rm W}(e\nu) = 79.86 \pm 0.16(\text{stat}) \pm 0.31(\text{syst}) \quad \text{GeV}. \tag{7}$$

Fig. 2 shows a summary of the direct measurements of $M_{\rm W}$, together with the indirect SM value from fit B (see section 6). In determining the world average value of the direct measurements of $M_{\rm W}$ the common systematic error of 85 MeV, due to uncertainties in the structure functions, is taken into account, giving

$$M_{\rm W} = 80.26 \pm 0.16 \quad \text{GeV}. \tag{8}$$

The traditional, but indirect, way of measuring the total width $\Gamma_{\rm W}$ is from the measurement of the ratio of the production of $\rm W \to \ell\nu$ to $\rm Z \to \ell\ell$, namely

$$R = \frac{\sigma_W}{\sigma_Z}.\frac{B(\rm W \to \ell\nu)}{B(\rm Z \to \ell\ell)} = \frac{\sigma_W}{\sigma_Z}.\frac{\Gamma(\rm W \to \ell\nu)}{\Gamma_{\rm W}}.\frac{\Gamma_{\rm Z}}{\Gamma(\rm Z \to \ell\ell)}. \tag{9}$$

To extract $\Gamma_{\rm W}$ requires input of $B(Z \to \ell\ell)$ and m_Z from LEP, σ_W/σ_Z from structure functions and the SM and also $M_{\rm W}$. The branching ratio $B(W \to \ell\nu)$ can then be extracted. If, in addition, the SM value of the $W \to \ell\nu$ partial width is assumed, then a measurement of R gives a value of $\Gamma_{\rm W}$. However, it should be stressed that the SM couplings of the W are invoked both in its production (σ_W/σ_Z depends on $u\bar{d}$ etc) and in its decay.

The recent indirect results are $\Gamma_{\rm W}$= 2.064 ± 0.085 GeV from CDF [14] ($W \to e\nu$ only) and $\Gamma_{\rm W}$= 2.044 ± 0.093 GeV from D0 [15] (e and μ). Combining these results with earlier UA1 [16] and UA2 data [17] gives[b]

$$\Gamma_{\rm W}(indirect) = 2.062 \pm 0.059 \quad \text{GeV}. \tag{10}$$

Recently, a direct measurement of $\Gamma_{\rm W}$, essentially free of SM assumptions, has been performed by the CDF collaboration [18], from a study of the high energy tail of the M_T^W distribution ($M_T^W > 110$ GeV), giving

$$\Gamma_{\rm W}(direct) = 2.11 \pm 0.32 \quad \text{GeV}. \tag{11}$$

The direct and indirect values are compatible with each other and with the SM value of $\Gamma_{\rm W}^{SM} = 2.077 \pm 0.014$ GeV, leaving room for a possible extra component from new physics of $\delta\Gamma_{\rm W}^{NEW} < 109$ MeV at the 95% c.l.

4. Measurement of the Z parameters

4.1. Z boson variables

The $Zf\bar{f}$ vertex can be described by effective vector ($v_{\rm f}$) and axial-vector ($a_{\rm f}$) couplings such that, to a very good approximation, the Born level formulae are retained. It is useful to define the *polarisation parameter*

$$A_{\rm f} = \frac{2v_{\rm f}a_{\rm f}}{(v_{\rm f}^2 + a_{\rm f}^2)} \quad . \tag{12}$$

The cross section for $e^+e^- \to f\bar{f}$, close to the Z pole, may be written in terms of the Z mass m_Z and total width Γ_Z as

$$\sigma_f(s) = \sigma_f^0 \frac{s\Gamma_Z^2}{(s - m_Z^2)^2 + s^2\Gamma_Z^2/m_Z^2} \quad + \gamma + (\gamma/Z)_{interf.} \ , \tag{13}$$

where the pole cross section σ_f^0 is defined as

$$\sigma_f^0 = \frac{12\pi\Gamma_{\rm e}\Gamma_{\rm f}}{m_Z^2\Gamma_Z^2} \quad . \tag{14}$$

[b] From [15], where the UA1, UA2 and CDF values are updated, using recent and consistent values for $B(Z \to \ell^+\ell^-)$ and $\Gamma(W \to \ell\nu)$ etc.

Here, $\Gamma_{\rm f}$ is the partial width for $\rm Z \to f\overline{f}$, which in turn can be written as [c]

$$\Gamma_{\rm f} = \frac{G_{\rm F} m_{\rm Z}^3}{6\pi\sqrt{2}}(v_{\rm f}^2 + a_{\rm f}^2) f_{QCD} f_{QED} \quad . \tag{15}$$

The final-state QED correction factor is $\mathrm{f}_{QED} = 1 + 3\alpha \mathrm{Q}_f^2/4\pi$, whereas f_{QCD} is unity for leptons and $\mathrm{f}_{QCD} = 3(1 + \mathrm{c}_q \alpha_{\rm s}(m_{\rm Z})/\pi + ..)$ for quarks, with $\mathrm{c}_q \simeq 1$.

The formula for the cross-section $\sigma_f(\mathrm{s})$ is an effective Born level formula and must be convoluted with QED radiative corrections before comparing with experiment. The measured cross sections are for $\mathrm{e^+e^-} \to$ hadrons and $\mathrm{e^+e^-} \to \ell^+\ell^-$, for $\ell =$ e,μ,τ. The hadronic width $\Gamma_{\rm had}$ is the sum of Γ_q for q=u,d,s,c,b. The total Z width, assuming lepton universality, is $\Gamma_{\rm Z} = \Gamma_{\rm had} + 3\Gamma_l + \Gamma_{\rm inv}$, where $\Gamma_{\rm inv} = 3\Gamma_\nu$ in the SM.

The distribution of the angle θ of the outgoing fermion f, with respect to the incident $\mathrm{e^-}$ direction, is given at Born level by

$$\frac{d\sigma}{d\cos\theta} = 1 + \cos^2\theta + \frac{8}{3} A_{\rm FB} \cos\theta \quad . \tag{16}$$

The *forward-backward asymmetry* is defined as $A_{\rm FB}^{\rm f} = (\sigma_F - \sigma_B)/(\sigma_F + \sigma_B)$, where $\sigma_F(\sigma_B)$ are the cross sections in the forward (backward) directions. The *pole asymmetry* is defined as

$$A_{\rm FB}^{0,\rm f} = \frac{3}{4} A_{\rm e} A_{\rm f} \ . \tag{17}$$

Since $A_{\rm f}$ depends on the ratio $v_{\rm f}/a_{\rm f}$, a measurement of $A_{\rm FB}^{0,\rm f}$ depends on both $v_{\rm e}/a_{\rm e}$ and $v_{\rm f}/a_{\rm f}$. The effective couplings can also be written as

$$a_{\rm f} = I_3^{\rm f}\sqrt{\rho_{\rm f}} \ , \qquad \frac{v_{\rm f}}{a_{\rm f}} = 1 - 4|Q_{\rm f}|\sin^2\theta_{\rm eff}^{\rm f} \ , \tag{18}$$

where the mixing angle defined for leptons ($\sin^2\theta_{\rm eff}^{\rm lept}$) is used for reference. Those defined for quarks have small shifts due to SM plus any new physics [19].

4.2. Determination of the LEP energy

Final results on the analysis of the LEP energy calibration to determine E_{LEP} for the 1993 scan are now available [20]. Good accuracy on E_{LEP} is crucial in the determination of $m_{\rm Z}$ and $\Gamma_{\rm Z}$. A precise measurement (E_{LEP}^{pol}, with $\delta \mathrm{E}_{cms} \leq 0.8$ MeV) of the average circulating beam energy can be made using the technique of *resonant depolarisation*. However, the LEP energy varies with time, due to the Earth tides and other effects such as the temperature of the dipole magnets.

Since the RF frequency, and thus the orbit length, is fixed, stresses in the local rock structure result in changes to the position of the beams in the quadrupole magnets.

[c] All the formulae here are for m_f=0. In practice, finite mass terms are taken into account.

This changes the beam energy, since the effective dipole field changes. These energy changes can be tracked accurately using measurements of the horizontal beam orbit positions, x_{orb}. This is important because, for the 1993 scan, only about one-third of the off-peak fills were calibrated with resonant depolarisation. The off-peak energies used were at about 1.8 GeV above and below the Z peak, and are referred to as peak-2 (38 fills, 13 calibrated) and peak+2 (31 fills, 11 calibrated) respectively.

A *model* for E_{LEP} was developed, based on x_{orb}, together with correction terms from the magnetic dipole fields and temperatures, the RF cavity voltages as well as other factors [20]. A single normalisation parameter was used at each of the off-peak energy points, resulting in rms values of E_{LEP}^{model} - E_{LEP}^{pol} of 6.9 and 2.8 MeV respectively at peak-2 and peak+2, for those fills for which there were polarisation measurements. These values are of importance in determining the energy error on those fills with no polarisation measurement, since it is assumed that these follow the same distribution.

Taking into account all the uncertainties, the estimated errors on the cms energies are 2.0 and 1.5 MeV at peak-2 and peak+2 respectively. The correlation coefficient is 0.3; this is important as components which are fully correlated do not contribute to $\delta\Gamma_{\rm Z}$. The error on the peak energy, which does not need to be known as precisely, is 5.4 MeV. The final contributions of the LEP energy to the errors on $m_{\rm Z}$ and $\Gamma_{\rm Z}$, taking into account also the 1990-1992 data and also the 1994 (peak) data, are $\delta m_{\rm Z}(\mathrm{LEP}) \simeq \pm$ 1.5 MeV and $\delta\Gamma_{\rm Z}(\mathrm{LEP}) \simeq \pm$ 1.7 MeV.

The uncertainty of the LEP cms energy spread (55 $\pm$ 5 MeV) also gives rise to an error on $\Gamma_{\rm Z}$, amounting to $\pm$ 1.0 MeV.

4.3. LEP data on cross sections and lepton asymmetries

Since the start-up of LEP in 1989 there has been an increase in the delivered luminosity each year and the total number of Z decays analysed by the four experiments, up to the end of 1994, is 12.4 million hadronic and 1.3 million leptonic events. The results from data up to 1992 have been published, whereas those from the 1993 scan and from the $\simeq$ 60 pb^{-1} on-peak data in 1994 are preliminary.

To match these impressive statistics the systematic errors need to be well understood; this is indeed the case. The experimental error on the *luminosity*, which is determined from the $e^+e^- \rightarrow e^+e^-$ cross section at small angles, is now determined to a precision ranging from 0.08 to 0.15% by the four experiments. This requires knowledge of both the absolute and relative positions of the detectors at the 10-20 μm level; an impressive achievement.

The event selection efficiency for $\sigma(e^+e^- \rightarrow \mathrm{hadrons})$ is known at a precision of $\sim$ 0.1%. The lepton cross section efficiencies are less well determined, but these measurements are still dominated by statistical errors, as are the errors on $A_{\rm FB}^{\ell}$.

The theoretical error on the luminosity has improved significantly since the 1994 ICHEP in Glasgow, when the error was $\delta\mathcal{L}/\mathcal{L}(\mathrm{theory}) = 0.25$ %. New calculations, using BHLUMI V4 [21], now include $\mathcal{O}(\alpha^2 L^2)$ terms (where L denotes the leading log term), as well as improved treatment of the γ-Z interference contributions. The current error is $\delta\mathcal{L}/\mathcal{L}(\text{theory}) = 0.16$ %, and it is expected that this will be reduced to 0.11% by the end of 1995.

For the purposes of combining the LEP data each experiment provides the results of a fit to their cross section and asymmetry data in terms of 9 variables. These are chosen to have small experimental correlations and are m_Z, Γ_Z, σ^0_h, R_ℓ and $A^{0,\ell}_{FB}$ (ℓ = e,μ,τ) The common errors from the LEP energy scale and energy spread, as well as that from the theoretical error on $\delta\mathcal{L}/\mathcal{L}$, are taken into account. The results of a 9 parameter fit to the combined LEP data are given in table 1. If *lepton universality* is imposed (evidence for this is discussed below), then there are 5 variables. The results of the fit to the combined LEP data are given in table 2. It can be seen from tables 1 and 2 that the Z mass and width are determined to δm_Z= 2.2 MeV and $\delta\Gamma_Z$= 3.2 MeV respectively. These are impressive accuracies. The new energy scan in 1995 will improve these even further.

Table 1. Results and correlation matrix of the 9 parameter fit to the LEP data. The χ^2/df of the average is 36.1/27.

quantity	value	error	m_Z	Γ_Z	σ^0_h	R_e	R_μ	R_τ	$A^{0,e}_{FB}$	$A^{0,\mu}_{FB}$	$A^{0,\tau}_{FB}$
m_Z(GeV)	91.1885	0.0022	1.00	-0.08	0.02	0.03	-0.02	-0.01	0.02	0.07	0.04
Γ_Z(GeV)	2.4963	0.0032		1.00	-0.12	-0.01	0.00	0.00	0.00	0.00	0.00
σ^0_h(nb)	41.488	0.078			1.00	0.08	0.12	0.08	0.01	0.00	0.00
R_e	20.797	0.058				1.00	0.08	0.03	-0.06	0.01	0.01
R_μ	20.796	0.043					1.00	0.06	0.00	0.01	0.00
R_τ	20.813	0.061						1.00	0.00	0.00	0.01
$A^{0,e}_{FB}$	0.0157	0.0028							1.00	-0.04	-0.02
$A^{0,\mu}_{FB}$	0.0163	0.0016								1.00	0.07
$A^{0,\tau}_{FB}$	0.0206	0.0023									1.00

Other quantities can be derived from these 9 or 5 parameter fits. Some of these are shown in table 3. In addition, Γ_{inv}/Γ_l= 5.956 $\pm$ 0.031 can be extracted, and when combined with the SM ratio Γ_ν/Γ_l = 1.991 $\pm$ 0.001, gives the number of light neutrinos N_ν = 2.991 $\pm$ 0.016.

4.4. Other data on lepton couplings

For the purposes of testing lepton universality and also for the global electroweak fits discussed in section 6 , the τ-polarisation and the left-right asymmetry A_{LR} from SLC/SLD are utilised. These are discussed in detail in [22] [23]. Fits to the angular distribution of the τ-polarisation give values for both A_e and A_τ, which are essentially uncorrelated. The current LEP average values are

$$A_e = 0.1390 \pm 0.0089 \qquad A_\tau = 0.1418 \pm 0.0075 \quad . \tag{19}$$

Table 2. Results and correlation matrix of the 5 parameter fit to the LEP data. The χ^2/df of the average is 39.3/31.

quantity	value	error	m_Z	Γ_Z	σ_h^0	R_ℓ	$A_{FB}^{0,\ell}$
m_Z(GeV)	91.1884	0.0022	1.00	-0.08	0.02	0.00	0.08
Γ_Z(GeV)	2.4963	0.0032		1.00	-0.12	-0.01	0.00
σ_h^0(nb)	41.488	0.078			1.00	0.15	0.01
R_ℓ	20.788	0.032				1.00	0.00
$A_{FB}^{0,\ell}$	0.0172	0.0012					1.00

Table 3. Quantities derived from the 9 and 5 parameter fits.

Without Lepton Universality		With Lepton Universality	
Γ_e(MeV)	83.92 ± 0.17	Γ_l(MeV)	83.93 ± 0.14
Γ_μ(MeV)	83.92 ± 0.23	Γ_{had}(MeV)	1744.8 ± 3.0
Γ_τ(MeV)	83.85 ± 0.29	Γ_{inv}(MeV)	499.9 ± 2.5

Assuming lepton universality these can be combined to give

$$A_\ell = 0.1406 \pm 0.0057 \quad . \tag{20}$$

The high values of longitudinal polarisation ($P_e \simeq 80$ %) achieved at the SLC have allowed the SLD experiment to make an extremely precise measurement of

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = A_e \quad , \tag{21}$$

where $\sigma_L(\sigma_R)$ is the total cross-section for a left (right) handed polarised incident electron beam. Combining all the data from 1992-5 gives a preliminary value of

$$A_e = 0.1551 \pm 0.0040, \qquad \sin^2\theta_{eff}^{lept} = 0.23049 \pm 0.00050, \tag{22}$$

which is compatible with the less precise value of A_e from τ decay at 1.7σ.

4.5. Lepton universality

The data from the leptonic partial decay widths, forward-backward asymmetries, τ-polarisation (A_τ and A_e) and A_{LR} have been used to fit to v_ℓ and a_ℓ (ℓ=e,μ, τ) and thus to test *lepton universality*. The results are shown in Fig. 3, where it can be seen that the data are consistent with the universality hypothesis. The signs are plotted taking $a_e < 0$. Using this convention (this is justified from ν-electron scattering results [24]), the signs of all couplings are uniquely determined from LEP data alone.

4.6. γ-Z interference term

In the fits described above the γ-Z interference term for the hadronic cross-section

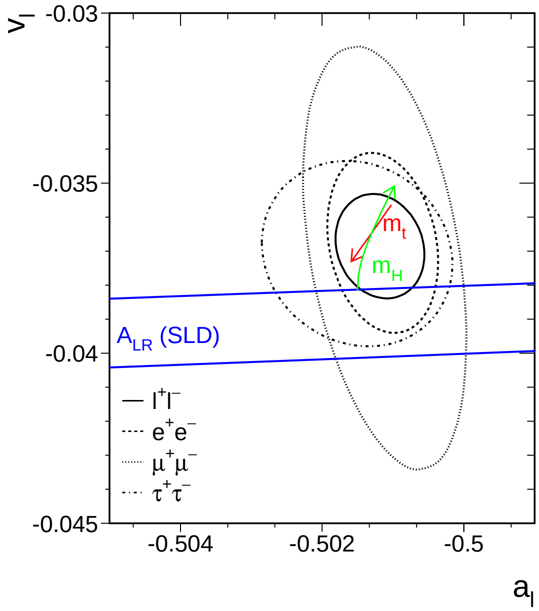

Fig. 3. The 68% probability contours in the vector axial-vector plane from LEP measurements. The solid contour results from a fit assuming lepton universality. Also shown is the one standard deviation band resulting from the $A_{\rm LR}$ measurement of SLD. The lines with arrows correspond to the SM prediction for $m_{\rm t} = 180 \pm 12$ GeV and $60 \leq m_{\rm H}$ [GeV] ≤ 1000 and point in the direction of increasing $m_{\rm t}$ and $m_{\rm H}$.

is fixed to its SM value ($\mathrm{j}_{\rm had}^{\rm tot} = 0.22 \pm 0.03$).[d] If this term is left free in the fits, then the error on $m_{\rm Z}$ increases significantly above 2.2 MeV, and is very strongly correlated to $\mathrm{j}_{\rm had}^{\rm tot}$. The error on $\Gamma_{\rm Z}$ is however increased only by a small amount to $\delta\Gamma_{\rm Z} = 3.4$ MeV. A fit to the combined LEP data gives

$$\delta m_{\rm Z} = \pm 6.2 \mathrm{MeV}, \qquad \mathrm{j}_{\rm had}^{\rm tot} = 0.18 \pm 0.34 \quad . \tag{23}$$

The errors on both these terms can be reduced by the inclusion of TOPAZ data from Tristan [26] at $\sqrt{s}$= 57.8 GeV, giving

$$\delta m_{\rm Z} = \pm 4.2 \mathrm{MeV}, \qquad \mathrm{j}_{\rm had}^{\rm tot} = 0.15 \pm 0.21 \quad . \tag{24}$$

The additional physics which is perhaps most likely to influence these parameters is from a possible Z' boson. However, taking into account the present experimental limits on the usual Z' models (η,ψ, L-R etc), the expected contributions to $\mathrm{j}_{\rm had}^{\rm tot}$ are rather small compared to the SM value. Furthermore, if such a Z' boson were to be discovered in the future, the corrections to $m_{\rm Z}$ and $\Gamma_{\rm Z}$ could be computed. It thus appears reasonable to use the errors on $m_{\rm Z}$ and $\Gamma_{\rm Z}$ obtained with $\mathrm{j}_{\rm had}^{\rm tot}$ fixed to its SM

[d] The notation here is that of the S-matrix formalism [25].

value (table 2).

5. Heavy flavour measurements at LEP and SLC

It is of intrinsic interest to extract the Z couplings to individual quark flavours, in contrast to the results described in section 4 which are summed over 5 flavours. In addition, the quantity

$$R_{\mathrm{b}} = \frac{\Gamma_{\mathrm{b}}}{\Gamma_{\mathrm{had}}} \tag{25}$$

is of particular interest. The propagator effects for the t-quark and Higgs, as well as QCD effects, largely cancel in this ratio. However, there are significant SM vertex corrections from tWb couplings. These are essentially independent of m_{H} and lead to a decrease of Γ_{b} with increasing m_{t}, rather than an increase as for the other quark partial-widths. Furthermore, R_{b} is sensitive to physics beyond the SM (eg. from light $\tilde{\mathrm{t}}$,$\tilde{\chi}$).

Extracting relatively pure samples of events corresponding to individual quark flavours is far from easy. Measurements exist for both c and b quarks, which can be separated from light (u,d,s) quarks , and from each other, using their characteristic properties (see table 4).

Table 4. Some properties of B hadrons and D mesons.

	B	D^+	D^0
lifetime (ps)	1.6	1.0	0.4
$< x_E = E_{had}/E_{beam} >$	0.7	0.5	0.5
decay charged multiplicity	5.5	2.2	2.2

The measured quantities are R_{c} and R_{b}, as well as the forward-backward asymmetries. The main selection criteria (*tags*) are as follows :-

- **c-quarks:** D,D* mesons plus lifetime and lepton tags. The harder momentum fraction in direct c compared to b→c is also used. For $A^{\mathrm{c}}_{\mathrm{FB}}$, the D/D* charges and the lepton charges, in semileptonic decays, are used to distinguish c from $\bar{c}$.

- **b-quarks:** lifetime and lepton tags. For $A^{\mathrm{b}}_{\mathrm{FB}}$ the lepton charge is used (evaluating the contributions from b→ ℓ, b→c→ ℓ, b→ $\bar{c} \to \ell$), as is also the jet-charge for a specific hemisphere with respect to the thrust-axis $Q_{hemi} = \sum |p^i_{||}|^{\kappa} Q_i / \sum |p^i_{||}|^{\kappa}$, where $p_{||}$ is the momentum component of a hadron with charge Q_i parallel to the thrust axis. The power κ is optimised for sensitivity. For R_{b} the most accurate results are from double-tag methods, as discussed below.

The main background in the tagged c-quark sample is from b-quarks and is roughly 20 ± 2 % per hemisphere. In the b-quark sample the c-quark background dominates

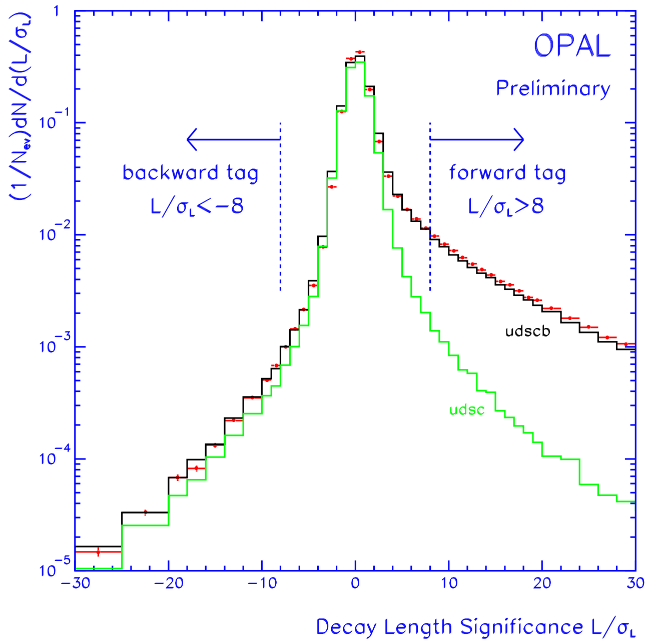

Fig. 4. Distribution of decay length significance for secondary vertices from the OPAL experiment.

and is roughly 5 $\pm$ 0.5 % per hemisphere. In addition, there are smaller backgrounds from u,d and s-quarks. An example of data used for a lifetime tag can be seen in fig. 4, which shows the distribution of the decay length significance per hemisphere, for the OPAL experiment. The clear excess for positive (forward) decay lengths due to b-quarks can be seen; the backward direction is used to control systematic effects. ALEPH, DELPHI and SLD use a somewhat different approach, which involves constructing a probability that a group of tracks contains some significant lifetime information.

The main systematic uncertainties arise from:-

- the fraction of D^*, D^+, D_s, Λ_c etc in $c\bar{c}$ events (particularly important for R_b)
- b and c hadron lifetimes
- charm decay modes
- fraction of $g \rightarrow c\bar{c}, b\bar{b}$ in hadronic Z events
- semi-leptonic decay models and branching ratios
- light quark fragmentation models
- correlations between hemispheres for double-tags

The results for A^c_{FB} and A^b_{FB} are given in figs. 5 and 6 respectively. The results for R_c are given in fig. 7 and those for R_b in fig. 8. It should be stressed that most of the heavy quark results are still preliminary.

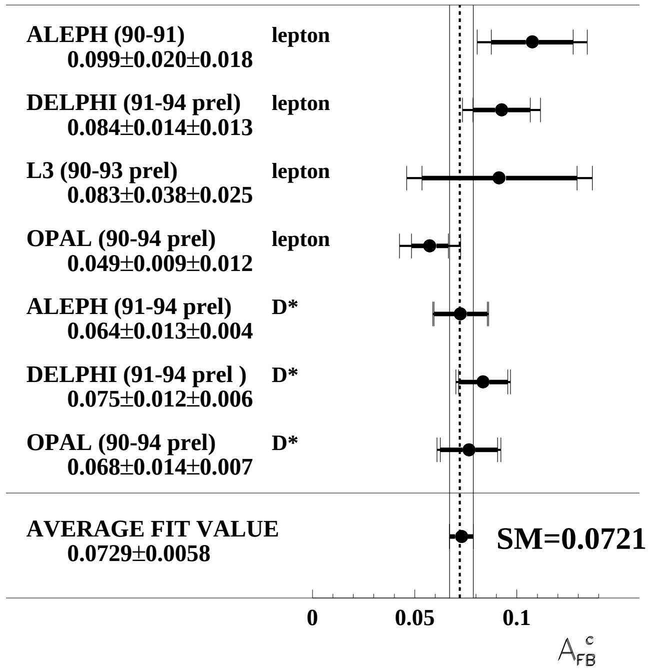

Fig. 5. Results on $A_{\rm FB}^{\rm c}$. The individual numbers are those quoted by the experiments. The plotted values have been transformed, if necessary, to correspond to the pole-asymmetry $A_{\rm FB}^{0,\,\rm c}$. The average fit value is that for $A_{\rm FB}^{0,\,\rm c}$.

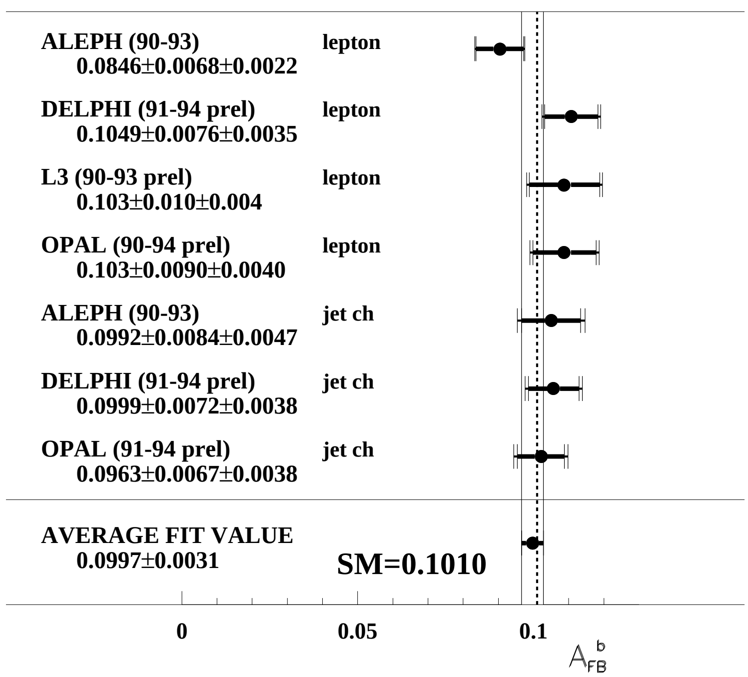

Fig. 6. Results on $A_{\rm FB}^{\rm b}$. The individual numbers are those quoted by the experiments. The plotted values have been transformed, if necessary, to correspond to the pole-asymmetry $A_{\rm FB}^{0,\,\rm b}$. The average fit value is that for $A_{\rm FB}^{0,\,\rm b}$.

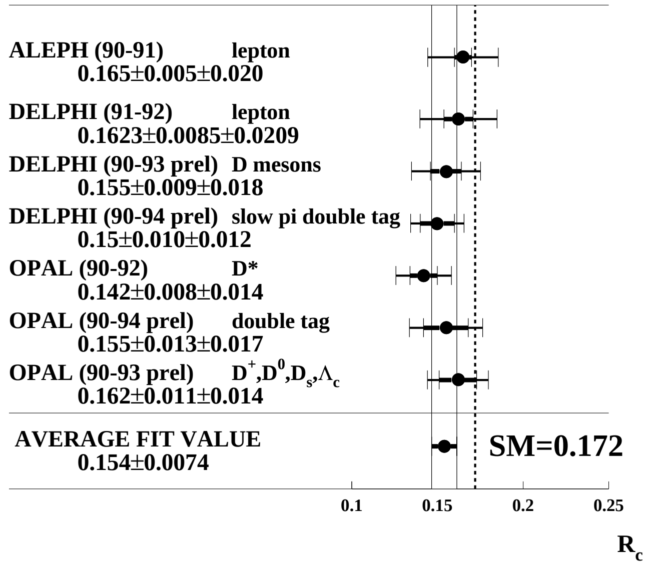

Fig. 7. Results on R_c, together with the average fit value.

5.1. *Combining the heavy flavour results*

The combination has been carried out by a LEP/SLD working group [27]. Each experiment provides, for each measurement, a complete breakdown of the systematic errors, adjusted if necessary to agreed meanings of these errors. Direct measurements of $\mathrm{A_b}$ and $\mathrm{A_c}$ by SLD [23], obtained by measuring $A_{\mathrm{FB}}^{\mathrm{b}}$ and $A_{\mathrm{FB}}^{\mathrm{c}}$ with a polarised beam, are also included. A multi-parameter fit is then performed to get the best overall values of R_{b},R_{c},$A_{\mathrm{FB}}^{0,\mathrm{b}}$, $A_{\mathrm{FB}}^{0,\mathrm{c}}$, $\mathrm{A_b}$, and $\mathrm{A_c}$, plus their covariance matrix. The results of a fit to both the LEP and SLD data are given in table 5.

The LEP values of $A_{\mathrm{FB}}^{0,\mathrm{b}}$, $A_{\mathrm{FB}}^{0,\mathrm{c}}$ and $A_\ell = 0.1464 \pm 0.0039$ (from $A_{\mathrm{FB}}^{0,\ell}$ and P_τ) can be used to give $\mathrm{A_b} = 0.910 \pm 0.037$ and $\mathrm{A_c} = 0.660 \pm 0.056$. Combining the LEP ($A_{\mathrm{FB}}^{0,\mathrm{b}}$, $A_{\mathrm{FB}}^{0,\mathrm{c}}$ and A_ℓ) and SLD ($\mathrm{A_b}$, $\mathrm{A_c}$ and A_{e}) results gives $\mathrm{A_b} = 0.871 \pm 0.027$ and $\mathrm{A_c} = 0.635 \pm 0.046$, to be compared to the SM values of 0.935 and 0.667 respectively (for $m_{\mathrm{t}} = 180$ GeV and $m_{\mathrm{H}} = 300$ GeV).

Since the results for R_{b} and R_{c} are found to be somewhat different to the SM predictions it is worth examining these measurements in a little more detail.

5.2. *Measurement of R_{b}*

The most accurate measurements all employ a double tag method. This involves determining the jet axis of the event (*thrust-axis*) and then employing a lifetime or leptonic tag to each hemisphere to determine

Table 5. Results of fits to the LEP and SLD heavy flavour data, plus the correlation matrix. The χ^2/df of the average is 55.1/(74-9), a probability of 80%.

quantity	value	error	R_{b}	R_{c}	$A_{\mathrm{FB}}^{0,\,\mathrm{b}}$	$A_{\mathrm{FB}}^{0,\,\mathrm{c}}$	$\mathcal{A}_{\mathrm{b}}$	$\mathcal{A}_{\mathrm{c}}$
R_{b}	0.2219	0.0017	1.000	−.345	.005	.055	−.068	.046
R_{c}	0.1540	0.0074		1.000	.084	−.063	.074	−.061
$A_{\mathrm{FB}}^{0,\,\mathrm{b}}$	0.0997	0.0031			1.000	.109	.062	−.025
$A_{\mathrm{FB}}^{0,\,\mathrm{c}}$	0.0729	0.0058				1.000	−.018	.073
$\mathcal{A}_{\mathrm{b}}$	0.841	0.053					1.000	.074
$\mathcal{A}_{\mathrm{c}}$	0.606	0.090						1.000

$$\begin{aligned} \mathrm{N}_t &\simeq 2\,\mathrm{N}_{had} R_{\mathrm{b}} \epsilon_b && = \text{number of hemispheres with a tag} \\ \mathrm{N}_{tt} &\simeq \mathrm{N}_{had} R_{\mathrm{b}} \epsilon_b^2 && = \text{number of events with 2 tags,} \end{aligned}$$

where N_{had} is the number of hadronic Z candidates and the tag efficiency (which is typically $\epsilon_b \sim 20\%$ for the lifetime and $\sim 5\%$ for the lepton tag respectively) can be determined *directly* from the data using these equations.

In practice there are significant complications from:-

- *backgrounds*, typically $\sim 0.5\%$. The main background is from $\mathrm{Z} \to \mathrm{c\bar{c}}$ which means that R_{b} is correlated with R_{c}.

- *hemisphere correlations*. These arise from QCD effects (eg the b and $\bar{b}$ end up in the same hemisphere), primary vertex (eg errors wrongly estimated), back-to-back 'holes' in the detector acceptance etc. These lead to an error $\sim 0.5\%$.

These (plus other) corrections are found using both the data and Monte Carlo. It is worth stressing that the Monte Carlo programs have been the subject of much effort and that much experimental input has been used to make them as reliable as possible.

The combined LEP/SLD value of $R_{\mathrm{b}} = 0.2205 \pm 0.0016$ ($R_{\mathrm{c}} = 0.172$) thus has a relative precision of about 0.7%. A breakdown of the common systematic errors has been made from a simplified, and thus approximate, computation. This is meant purely as a guide in understanding the current experimental status. The double-tag results from ALEPH, DELPHI, OPAL and SLD account for about 80% of the total weight, with the lepton and event-shape analyses accounting for the remaining 20%. The combined statistical error of all the measurements is about 0.0008 and that from the internal experimental systematics (track resolution, detection efficiencies of leptons etc) is about 0.0007. The error due to common systematics is about 0.0013. The largest common systematic errors are from uncertainties on the fractions of D^+, D^0 and D_s (0.0007), the D decay multiplicity (0.0005), the D-meson lifetimes (0.0003), the light quark fragmentation properties (0.0003), and the branching ratio $\mathrm{D}\to\mathrm{K}^0$ (0.0003). In total more than 20 sources of error to R_{b} are considered.

Thus R_{b} is systematics limited. Each experiment chooses cuts to optimise its overall error. Although considerable effort [27] has been made to ensure common input

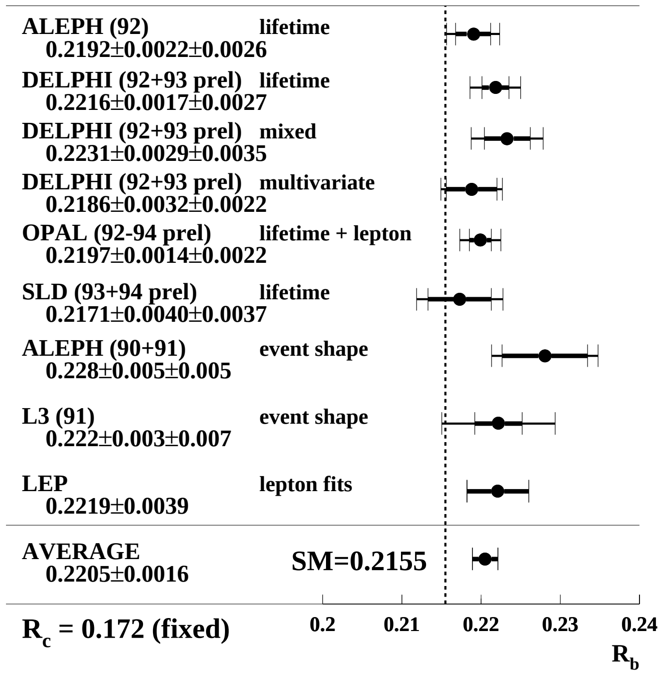

Fig. 8. Results on R_b, together with the average fit value for $R_c = 0.172$. The first 7 results use a double-tag method.

parameters, and of the definition of systematic errors between experiments, it should be stressed that systematic errors do not, in general, have the same precise meaning in terms of confidence levels as statistical errors. However, the total systematic error on R_b is made up from many components, so it is to be expected that some cancellations in the signs of the contributions will occur (the components are added in quadrature). It is thus unlikely that they could all be "wrong" in the sense that they shift R_b in the same direction.

The background of c in b samples can, to some extent, be reduced by making harder cuts on the lifetime or lepton properties used. However, such cuts might well increase the error from hemisphere correlations. Thus increased statistics alone do not guarantee a more accurate result.

5.3. Measurement of R_c

Tagging charm quarks with high efficiency and purity is unfortunately difficult. The cleanest tag is to use c$\rightarrow$ $D^{*+} \rightarrow D^0\pi^+(D^0 \rightarrow K^-\pi^+)$, but this tags only about 0.5% of c-decays, so is statistically limited. Other D^0 modes, which are somewhat less clean, can also be used, as can leptonic c-quark decays. However, the overall charm tag efficiency is low compared to that for b-quarks. This means that a double tag method, using tight tags, is not feasible. However, there are two new analyses which use a “slow” π tag (the π in the D^* decay has a small p_T) in a double tag. This tag is rather loose because there is a considerable background at low p_T from fragmentation processes. OPAL apply a tight tag in one hemisphere and, if successful, look for a slow π tag in the opposite hemisphere. This allows a direct measurement of Br(c$\rightarrow$ $D^\star$)= 0.237 $\pm$ 0.027, a result compatible with that obtained at low ($\sqrt{s} \sim 10$ GeV) energy. This is important because many analyses rely on assuming that Br(c$\rightarrow$ $D^\star$) is essentially energy independent. DELPHI employs a full double tag method using slow π's, but the increased statistics are at the expense of more background.

The relative precision of the average value of $R_{\rm c}$ (=0.1540 $\pm$ 0.0074) is 4.8%. A simplified method can again be used to gauge the various contributions to the total error. About 60% of the error weight comes from the combined statistical and internal experimental systematic errors, with the remainder from the common systematic errors. This leaves scope for improvement with increased statistics. The largest common systematic errors are from the normalisation error using the charm production fractions measured at low energies (0.0030), br(c$\rightarrow$ ℓ) (0.0025) and the b-hadron lifetimes (0.0023).

As discussed above, the determinations of $R_{\rm b}$ and $R_{\rm c}$ are correlated. Fig. 9 shows various confidence level contours in the $R_{\rm b}$, $R_{\rm c}$ plane. It can be seen that the experimental result is only consistent with the SM at about the 0.1% level. The sum of $R_{\rm b} + R_{\rm c}$ = 0.3759 $\pm$ 0.0070, compared to the SM value of 0.3879 ($m_{\rm t}$=180 GeV, $m_{\rm H}$=300 GeV); a difference of 1.7σ. The error in the sum is dominated by that of $R_{\rm c}$. However, the naive temptation to assume c-quark events have been wrongly classified as b-quarks should be resisted, since the overlap of events in the most precise $R_{\rm b}$ and $R_{\rm c}$ analyses is small. Nevertheless, there are sources of systematic error (eg. the charm multiplicity) which affect both analyses.

The asymmetries $A_{\rm FB}^{0,\rm b}$ and $A_{\rm FB}^{0,\rm c}$ are more weakly correlated, and both the pole asymmetries, and their energy dependence (see fig. 10) are both compatible with the SM.

5.4. Z $\rightarrow q\bar{q}\gamma$ radiative events

Since the coupling of a γ to a quark is $\propto Q_q^2$, the number of Z events which result in a $q\bar{q}\gamma$ final state is $N_{q\bar{q}\gamma} \simeq 4R_{\rm u} + 4R_{\rm c} + R_{\rm d} + R_{\rm s} + R_{\rm b}$. The number of hadronic Z events $N_{q\bar{q}} \simeq R_{\rm u} + R_{\rm c} + R_{\rm d} + R_{\rm s} + R_{\rm b}$. Measurements of isolated photon production have been made by all the LEP experiments and the observed yield compared to matrix element calculations of $\mathcal{O}(\alpha\alpha_s)$. The fraction of radiative events can be used to extract [28] $(R_{\rm u} + R_{\rm c})/2$ = 0.147 $\pm$ 0.019 (SM=.1724). Alternatively, this result can be expressed as $(R_{\rm d} + R_{\rm s} + R_{\rm b})/3$ = 0.235 $\pm$ 0.013 (SM=.2184). This result suffers

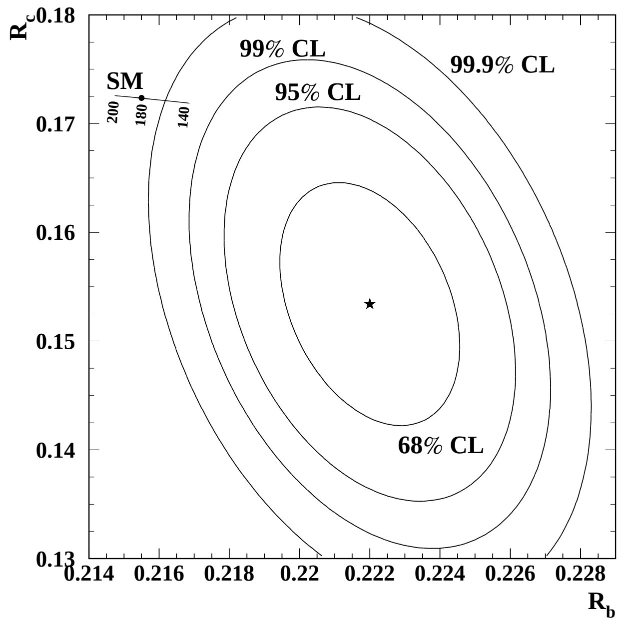

Fig. 9. Confidence level contours in the $R_{\rm b}$, $R_{\rm c}$ plane.

from different systematic errors to those of the direct measurements of $R_{\rm b}$ and $R_{\rm c}$.

6. Global electroweak fits

6.1. Determinations of $\sin^2\theta_{\rm eff}^{\rm lept}$

The values of $\sin^2\theta_{\rm eff}^{\rm lept}$ determined by the various methods are shown in Fig. 11. In addition to the measurements already described there is also a determination using Q_{FB}, the inclusive charge-asymmetry in hadronic Z decays, with new measurements from ALEPH and OPAL. All the results are statistically compatible and have an impressive precision of $\pm$ 0.00028. It is an interesting question as to whether one expects agreement amongst these measurements. The various methods have different sensitivities to different couplings, so differences could be sensitive to different physics beyond the SM. The sensitivity of the heavy quark asymmetries $A_{\rm FB}^{\rm b}$ and $A_{\rm FB}^{\rm c}$ to $\sin^2\theta_{\rm eff}^{\rm lept}$ is mainly through $A_{\rm e}$ rather than $A_{\rm b}$ and $A_{\rm c}$ (see 12,17 and 18), which are rather insensitive to variations of the SM parameters. The range of $\sin^2\theta_{\rm eff}^{\rm lept}$ extracted from either $A_{\rm FB}^{\rm b}$ or $A_{\rm FB}^{\rm c}$ is $\sim 2\ 10^{-5}$, for variations within the SM of $100 < m_{\rm t} < 200$ GeV and $60 < m_{\rm H} < 1000$ GeV.

6.2. Standard Model Constraints

The precise electroweak measurements performed at LEP can be used to check the validity of the Standard Model and, within its framework, to infer valuable infor-

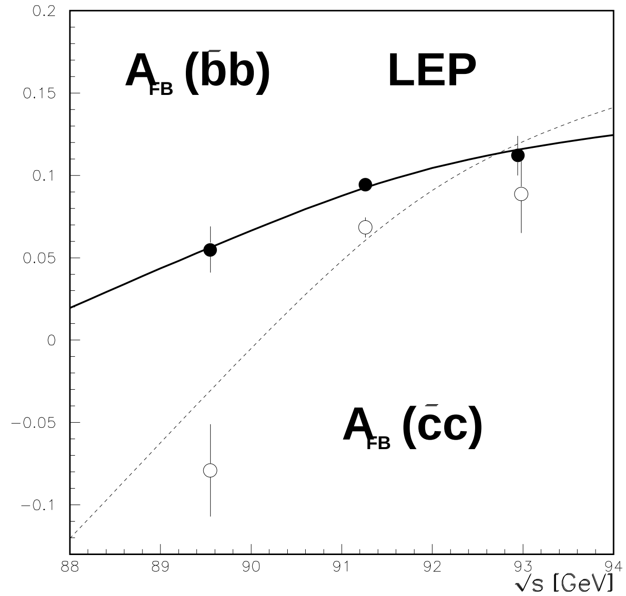

Fig. 10. Energy dependence of the heavy quark forward-backward asymmetries.

mation about its basic parameters. Their accuracy makes them sensitive to the top quark mass, m_t, and to the mass of the Higgs boson, m_H, through the loop corrections. The leading top quark dependence is quadratic and allows a determination of m_t. The main dependence on m_H is logarithmic and therefore, with the present data accuracy, the constraints on m_H are much weaker.

The various measurements used in the SM fits are summarised in Table 6. Table 7 shows the constraints obtained on m_t and $\alpha_s(m_Z)$ when fitting the measurements to the most recent SM calculations [29]. The estimated theoretical uncertainties are $\Delta m_t \leq 2$ GeV and $\Delta\alpha_s(m_Z) \leq 0.001$. An uncertainty in $1/\alpha(m_Z)$ of 0.09 (section 2) causes an error of 0.00023 on the SM prediction of $\sin^2\theta_{\rm eff}^{\rm lept}$ and an error of 4 GeV on m_t. The effect on the prediction for Γ_l is negligible.

The fitted values of m_t are compatible with the direct Tevatron measurement of $m_t = 180 \pm 12$ GeV. Fig. 12 shows a comparison of the direct versus indirect (from an SM fit to LEP and SLD data) determinations of M_W and m_t.

The value of $\alpha_s(m_Z)$ depends mainly on R_ℓ, Γ_Z and σ_h^0, and is in good agreement with event shape analyses at LEP ($\alpha_s(m_Z) = 0.123 \pm 0.006$ [30]). The strong coupling constant can also be determined from R_ℓ alone. Imposing $m_t = 180 \pm 12$ GeV as constraint, $\alpha_s(m_Z) = 0.126 \pm 0.005 \pm 0.002$ is obtained, where the second error corresponds to $60 \leq m_H$ [GeV] ≤ 1000.

The largest differences to the SM fit (see table 6) are for the quantities R_b ($+3.7\sigma$), R_c (-2.5σ) and A_{LR}(-2.5σ). If R_c is fixed to its SM value 0.172, the fitted value is R_b=0.2205 $\pm$ 0.0016, and the corresponding pull $+3.2\sigma$. If R_b and R_c are removed from the fit, the value of m_t increases by about 4 GeV, whereas that of $\alpha_s(m_Z)$ is essentially unchanged. The χ^2 decreases by 15.

If we were to attribute the excess in R_b to possible new physics, by defining $\Gamma_b =$

Table 6. Measurements used in SM fits, together with the corresponding SM fit value for Fit C of table 7 and the pull (the difference between the measurement and the fit divided by the measurement error).

	Measurement	SM fit	Pull
a) LEP			
m_Z [GeV]	91.1884 ± 0.0022	91.1882	0.1
Γ_Z [GeV]	2.4963 ± 0.0032	2.4973	-0.3
σ_h^0 [nb]	41.488 ± 0.078	41.450	0.5
R_ℓ	20.788 ± 0.032	20.773	0.5
$A_{FB}^{0,\ell}$	0.0172 ± 0.0012	0.0159	1.1
$\mathcal{A}_\tau$	0.1418 ± 0.0075	0.1455	-0.5
$\mathcal{A}_e$	0.1390 ± 0.0089	0.1455	-0.7
R_b	0.2219 ± 0.0017	0.2156	3.7
R_c	0.1543 ± 0.0074	0.1724	-2.5
$A_{FB}^{0,b}$	0.0999 ± 0.0031	0.1020	-0.7
$A_{FB}^{0,c}$	0.0725 ± 0.0058	0.0728	0.0
$\sin^2\theta_{eff}^{lept}$ ($\langle Q_{FB}\rangle$)	0.2325 ± 0.0013	0.23172	0.6
b) SLC			
$\sin^2\theta_{eff}^{lept}$ (A_{LR})	0.23049 ± 0.00050	0.23172	-2.5
R_b	0.2171 ± 0.0054	0.2156	0.3
$\mathcal{A}_b$	0.841 ± 0.053	0.935	-1.8
$\mathcal{A}_c$	0.606 ± 0.090	0.667	-0.7
c) $p\bar{p}$ and νN			
M_W [GeV]	80.26 ± 0.16	80.35	-0.5
$1 - M_W^2/m_Z^2$	0.2257 ± 0.0047	0.2237	0.4

Table 7. Results of SM fits for m_t and $\alpha_s(m_Z)$. No external constraint on $\alpha_s(m_Z)$ is imposed. The central values and the first errors quoted refer to $m_H = 300$ GeV. The second errors correspond to the variation $60 \leq m_H$ [GeV] ≤ 1000. The bottom part of the table lists derived results for $\sin^2\theta_{eff}^{lept}$, $1 - M_W^2/m_Z^2$ and M_W. The constraint from the direct measurement of m_t is not used in these fits.

	A) LEP	B) LEP + SLD	C) LEP + SLD + $p\bar{p}$ and νN data
m_t (GeV)	$170 \pm 10 \, ^{+17}_{-19}$	$180 \, ^{+8}_{-9} \, ^{+17}_{-20}$	$178 \pm 8 \, ^{+17}_{-20}$
$\alpha_s(m_Z)$	$0.125 \pm 0.004 \pm 0.002$	$0.123 \pm 0.004 \pm 0.002$	$0.123 \pm 0.004 \pm 0.002$
χ^2/d.o.f. (prob)	18/9 (3.5%)	28/12 (0.6%)	28/14 (1.4%)
$\sin^2\theta_{eff}^{lept}$	$0.23206 \pm 0.00028 \, ^{+0.00008}_{-0.00017}$	$0.23166 \pm 0.00025 \, ^{+0.00006}_{-0.00013}$	$0.23172 \pm 0.00024 \, ^{+0.00007}_{-0.00014}$
$1 - M_W^2/m_Z^2$	$0.2247 \pm 0.0010 \, ^{+0.0004}_{-0.0002}$	$0.2234 \pm 0.0009 \, ^{+0.0005}_{-0.0002}$	$0.2237 \pm 0.0009 \, ^{+0.0004}_{-0.0002}$
M_W (GeV)	$80.295 \pm 0.057 \, ^{+0.011}_{-0.019}$	$80.359 \pm 0.051 \, ^{+0.013}_{-0.024}$	$80.346 \pm 0.046 \, ^{+0.012}_{-0.021}$

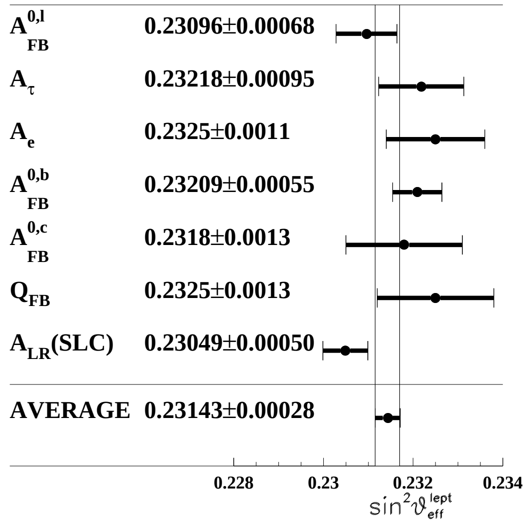

Fig. 11. Measurements of $\sin^2\theta_{\rm eff}^{\rm lept}$. The χ^2/df = 7.8/6, corresponding to a probability of 25%.

$\Gamma_{\rm b}$(SM) + $\Delta\Gamma_{\rm b}$, then a fit can be made to all the data, propagating this definition to all variables depending on $\Gamma_{\rm had}$. A repeat of Fit C in table 7 yields

$$\Delta\Gamma_{\rm b} = 11.7 \pm 3.8 \pm 1.4 \text{ MeV}, \ \alpha_{\rm s}(m_{\rm Z}) = 0.102 \pm 0.008 \ , \ m_{\rm t} = 181 \pm 8 \ ^{+17}_{-19} \text{ GeV}, \quad (26)$$

with a χ^2 of 18/13 df, compared to 28/14 for Fit C. Note that there is a strong correlation between $\Delta\Gamma_{\rm b}$ and $\alpha_{\rm s}(m_{\rm Z})$, so that the extra piece in $\Gamma_{\rm b}$ is accommodated by having a much lower value of $\alpha_{\rm s}(m_{\rm Z})$. This procedure could be extended to allow also a term $\Delta\Gamma_{\rm c}$, however the relative precision on $R_{\rm c}$ is much poorer and $\alpha_{\rm s}(m_{\rm Z})$ is not sensibly constrained in such a fit by electroweak data alone.

6.3. Extraction of heavy quark couplings

An alternative approach in trying to understand the possible implications of the heavy flavour results is to extract the quark couplings. The measurements used are $R_{\rm b} = \Gamma_{\rm b}/\Gamma_{\rm had}$ (which, using $\Gamma_{\rm had}$ from the lineshape, gives $v_{\rm b}^2$+ $a_{\rm b}^2$), $R_{\rm c}$ ($v_{\rm c}^2$ + $a_{\rm c}^2$), $A_{\rm e}$ from LEP/SLD ($v_{\rm e}/a_{\rm e}$), $A_{\rm FB}^{\rm b}$ ($v_{\rm b}/a_{\rm b}$, $v_{\rm e}/a_{\rm e}$), $\mathrm{A_b}$ ($v_{\rm b}/a_{\rm b}$), $A_{\rm FB}^{\rm c}$ ($v_{\rm c}/a_{\rm c}$, $v_{\rm e}/a_{\rm e}$) and $\mathrm{A_c}$ ($v_{\rm c}/a_{\rm c}$). The constraint $\alpha_{\rm s}(m_{\rm Z})$ = 0.123 ± 0.006 is imposed. The results are shown in fig. 13, together with the SM predictions.

It is interesting to note that the left-handed b-coupling ($v_{\rm b}$ + $a_{\rm b}$) is compatible with the SM value, whereas the right-handed b-coupling ($v_{\rm b}$ - $a_{\rm b}$) is larger in value. For the c-quark, the right-handed c-coupling agrees with the SM whereas the left-handed c-coupling is less than the SM prediction.

6.4. Fits to $m_{\rm H}$

In the fits discussed so far $m_{\rm H}$ has been held fixed. Figure 14 shows the observed value of $\Delta\chi^2 \equiv \chi^2 - \chi^2_{min}$ as a function of $m_{\rm H}$, when the Tevatron value of $m_{\rm t}$

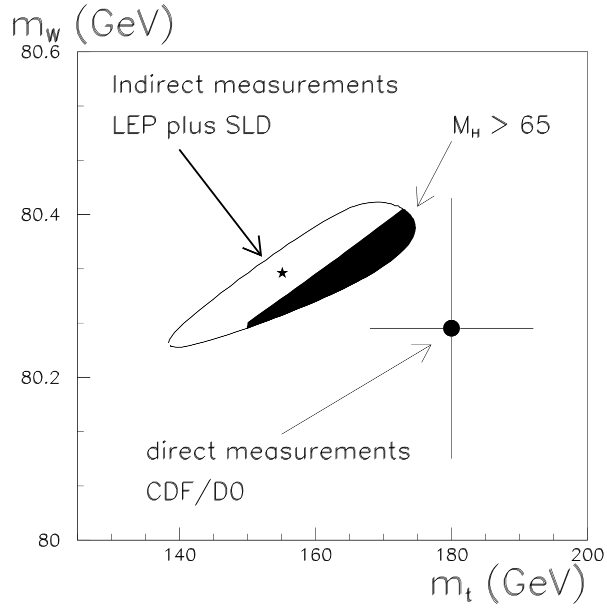

Fig. 12. A comparison of direct versus indirect determinations of m_t and M_W. The filled part of the contour is that compatible with $m_H \geq 65$ GeV from direct searches.

is used as an additional constraint in the fit. The observed $\Delta\chi^2$ curve exhibits a minimum for low values of m_H, with the minimum at 72 GeV (138 GeV) including (excluding) the measurements of R_b and R_c. The $\Delta\chi^2 = 2.7$ interval, approximately corresponding to a 95% confidence level upper limit, extends up to 500 GeV (750 GeV) including (excluding) the measurements of R_b and R_c. Again the results are sensitive to the uncertainty in $\alpha(m_Z)$; a shift of $+0.09$ in $\alpha(m_Z)^{-1}$, with other parameters held constant, shifts the Higgs mass upwards by about 40 GeV from 72 GeV.

Fig. 15 shows the 95% c.l. contours of m_t and m_H for a fit to the data used for Fit C, plus a contraint using the Tevatron top-quark mass measurement. The contours are given with and without the inclusion of R_b and R_c in the fit. The central values of these fits, plus a further fit in which the SLD measurement of A_{LR} is also removed, are given in table 8. The fitted value of m_H is thus rather sensitive to the inclusion of these measurements, which are 2.5σ or more from the SM fit values.

The expected precision from the current level of accuracy of these quantities can be gauged by performing SM fits to the data in which the measured values of all quantities are set to their SM values for m_t= 180 GeV, m_H = 100 GeV and $\alpha_s(m_Z)$= 0.123, but with their errors and correlations unchanged. The resulting fits give $m_t = 180^{+11}_{-10}$ GeV for the 3 fits of table 8, with m_H= 100^{+170}_{-64} GeV for the fit to all data, m_H= 100^{+174}_{-64} GeV if R_b and R_c are excluded, and m_H= 100^{+190}_{-67} GeV if also A_{LR} is excluded. The error on $\alpha_s(m_Z)$ in each case is $\simeq 0.0038$.

The contour plots shown in fig. 15 show that a heavy Higgs with $m_H > 1$ TeV cannot yet be excluded. Hence, although the data are now precise enough to have some sensitivity to m_H and indeed favour a relatively light Higgs, some caution is

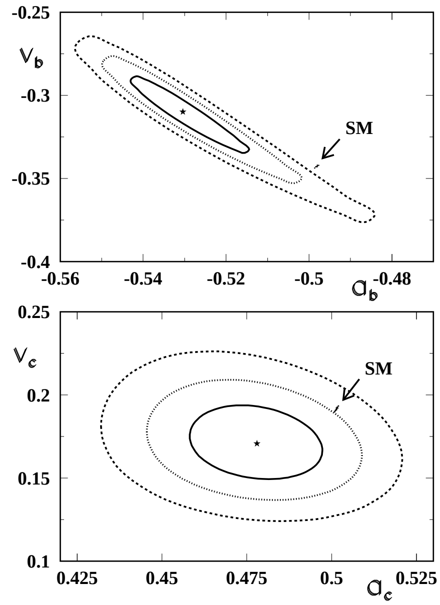

Fig. 13. Results of a fit to the b and c vector and axial-vector couplings. The contours are for the 68, 95 and 97.7 % confidence limits.

necessary in the interpretation of the results.

Table 8. Results of SM fits to m_t, m_H and $\alpha_s(m_Z)$. In the first column all data are used, in the second R_b and R_c are excluded and in the third also A_{LR} is excluded.

	all data	R_b and R_c excluded	R_b, R_c and A_{LR} excluded
m_t GeV	167^{+11}_{-8}	175^{+13}_{-12}	179^{+12}_{-12}
m_H GeV	72^{+148}_{-48}	138^{+282}_{-101}	365^{+643}_{-240}
$\alpha_s(m_Z)$	0.1216 ± 0.0037	0.1221 ± 0.0039	0.1247 ± 0.0042

7. Triple gauge vertex (TGV) couplings

Although, as discussed above, there are very precise measurements on the $Zf\bar{f}$ couplings, particularly for $f = \ell$, the accuracy on TGV couplings is relatively poor. Measurements of the TGV couplings are very important tests of the SM. Studies involve measurements of $q\bar{q} \to$ WW, Wγ, Zγ, WZ at hadron colliders (Tevatron, LHC) and $e^+e^- \to$ Zγ, WW at LEP (particularly LEP Phase 2). In the SM there are substantial cancellations between t- and u-channel diagrams (which involve only couplings of the bosons to fermions) and the s-channel three-boson couplings.

The general formalism is rather complicated (see eg. [31]) and the notation not unique! The WWV vertex (V=Z,γ) can be described by an effective Lagrangian $\mathcal{L}_{eff}^{WWV}$ which, assuming CP conservation, has 5 parameters ($g_1^Z, \kappa_Z, \kappa_\gamma, \lambda_Z$ and λ_γ). It is usual to define an anomalous coupling as the difference between the value of a

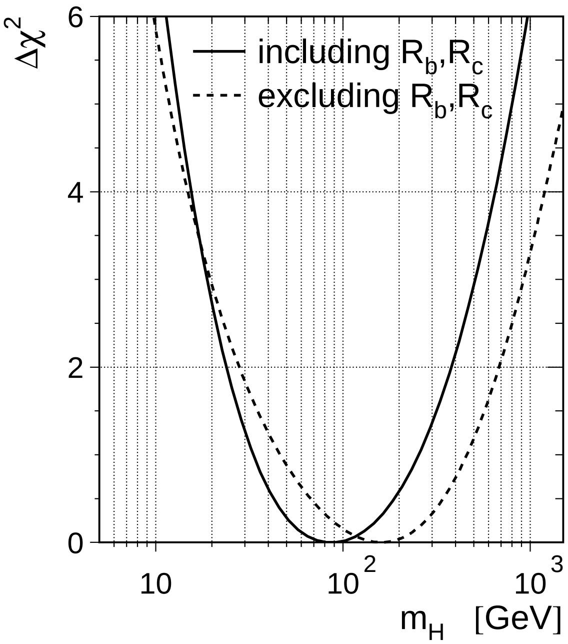

Fig. 14. $\Delta\chi^2 \equiv \chi^2 - \chi^2_{min}$ vs $m_{\rm H}$, with and without inclusion of measurements on $R_{\rm b}$ and $R_{\rm c}$.

parameter and its SM value at tree-level:-

$$\Delta g_1^Z = (g_1^Z - 1), \quad \Delta\kappa_Z = (\kappa_Z - 1), \quad \Delta\kappa_\gamma = (\kappa_\gamma - 1), \quad \lambda_Z, \quad \lambda_\gamma \ . \tag{27}$$

The W magnetic dipole and electric quadrupole moments are given by $\mu_W = \frac{e}{2M_{\rm W}}(1+\kappa_\gamma+\lambda_\gamma)$ and $Q^e_W = -\frac{e}{M^2_{\rm W}}(\kappa_\gamma - \lambda_\gamma)$ respectively. The effective Lagrangian for the ZγV vertex, $\mathcal{L}^{Z\gamma V}_{eff}$, depends on two parameters if CP consevation is assumed. These are h^V_3 and h^V_4, which are zero at tree-level.

Since the formalism is in terms of an effective Lagrangian, there are potential unitarity problems at large energy scales, so the above parameters (all of which are functions of some energy scale) must be dampened at large q^2 by a suitable form-factor behaviour. For example,

$$\Delta\kappa_V = \frac{\Delta\kappa_V(0)}{(1+q^2/\Lambda^2)^n}. \tag{28}$$

The scale Λ is related to the onset of new physics. It is customary to take n=2 for WWV and n=3(4) for h^V_3 (h^V_4). The expected size of the anomalous TGVs is, for an energy scale $\Lambda \sim 1$ TeV, $\mathcal{O}(\frac{M^2_{\rm W}}{\Lambda^2}) = 10^{-2}$. This is small on the scale of current, and indeed probable future, precision [31].

The only results involving W bosons so far are from hadron colliders. Both CDF [32] and D0 [33] have given limits on WWγ, from a study of p$\bar{\rm p}$ $\to$Wγ (W$\to \ell\nu$). Fig. 16 shows the results along with the unitarity limits for a scale $\Lambda_W = 1.5$ TeV. It is usual practice to choose the Λ scale such that it is just outside the experimental contours.

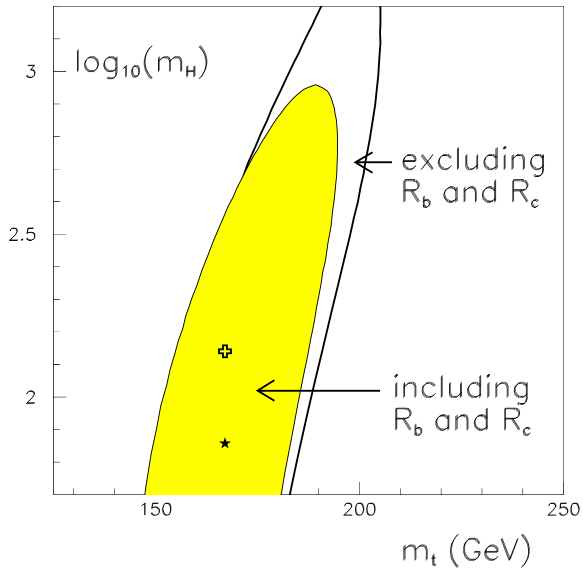

Fig. 15. 95% c.l. contour plots for $m_{\rm t}$ vs $m_{\rm H}$, with and without measurements of $R_{\rm b}$ and $R_{\rm c}$.

The current precision is approaching interesting levels; it is sufficient to exclude the values $\mu_W = Q^e_W = 0$, at the 99% c.l. So far no attempt has been made by the experiments to combine their results. The present limits are, $\mid \Delta\kappa_\gamma \mid \lesssim 1.7$ (λ_γ=0) and $\mid \lambda_\gamma \mid \lesssim 0.6$ ($\Delta\kappa_\gamma$=0), at the 95% c.l. and for $\Lambda = 1.5$ TeV.

CDF [34] and D0 [35] have also studied the final states $p\bar{p} \to$ WW (W$\to \ell\nu$) and $p\bar{p} \to$ WW,WZ (W$\to \ell\nu$, W,Z$\to$jj). The WW final state depends on the WWγ and WWZ couplings, whereas the WZ final state depends only on the WWZ coupling. Experimentally these final states have not been distinguished in the jj studies. Furthermore, the simplifying assumptions $\Delta\kappa_\gamma = \Delta\kappa_Z = \Delta\kappa$ and $\lambda_\gamma = \lambda_Z = \lambda$ can been made. At the 95% c.l. the current, and preliminary, limits for a form-factor scale of $\Lambda = 1.5$ TeV are $\mid \Delta\kappa \mid \lesssim 0.9$ (λ=0) and $\mid \lambda \mid \lesssim 0.6$ ($\Delta\kappa$=0) (see fig. 17).

The ZZγ couplings are determined by CDF [36] and D0 [37] from a study of $p\bar{p} \to$ Zγ (Z $\to \ell\ell$). The resulting limits are $\mid h^Z_{30} \mid \lesssim 1.8$ (h^Z_{40}=0), $\mid h^Z_{40} \mid \lesssim 0.5$ (h^Z_{30}=0), for a form-factor scale $\Lambda = 0.5$ TeV. The Z$\gamma\gamma$ limits are rather similar. There is also a limit from the L3 experiment [38] which gives $\mid h^Z_{30} \mid \lesssim 0.85$ (h^Z_{40}=0), $\mid h^Z_{40} \mid \lesssim 2.3$ (h^Z_{30}=0).

8. Conclusions

The precision electroweak data from LEP, SLD and the Tevatron have now reached levels of precision such that they are very sensitive to electroweak radiative corrections, allowing indirect measurements of the SM parameters. The direct and indirect measurements of $m_{\rm t}$ and $M_{\rm W}$ are compatible, which implies stringent limits on physics

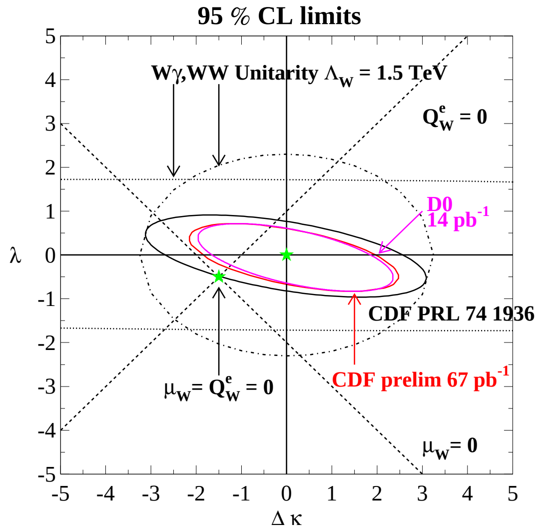

Fig. 16. 95% confidence level contours for $\Delta\kappa_\gamma$ against λ_γ from a study of Wγ final states. The unitarity constraints for a scale Λ_W = 1.5 TeV, and the lines μ_W=0 and Q^e_W=0, are also shown.

beyond the SM. New data from the 1995 LEP energy scan, from SLC on $A_{\rm LR}$, and from Fermilab and LEP on $M_{\rm W}$ (an error on $M_{\rm W} \sim 30$ MeV appears feasible eventually) will significantly improve the precision of these tests. Improved precision on $\alpha(m_{\rm Z})$, through further low energy e^+e^- measurements of R_{had}, is most desirable.

Significant data on testing the triple gauge boson vertices are now emerging from Fermilab. These studies will continue also at LEP 2 and LHC and will occupy us for the next 10 years or more.

The largest discrepancies with the SM are from $R_{\rm b}$ and $R_{\rm c}$. These are difficult measurements and the data are largely preliminary. While this disagreement is intriguing, more work is needed experimentally before a believable crack in the SM can be firmly established. At face value, the results would imply that the quark couplings are very different from their SM values. The results also have a subtle interplay with the value of $\alpha_{\rm s}(m_{\rm Z})$ extracted from $\Gamma_{\rm had}$.

These differences do not affect significantly the impressive success of the SM in the agreement of direct and indirect measurements of $m_{\rm t}$. The data favour a rather light Higgs mass ($\lesssim$ 150 GeV). However, it should be recalled that the errors are logarithmic in $m_{\rm H}$ and that a significant upper bound, below the 1 TeV level, is yet to be established.

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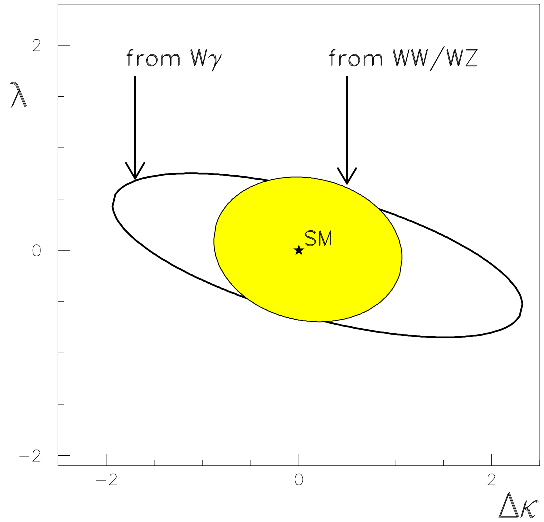

Fig. 17. 95% confidence level contours for $\Delta\kappa$ against λ, for a form-factor scale $\Lambda = 1.5$ TeV, from a study of WW and WZ final states and also from Wγ studies, from the CDF and D0 experiments. These contours are taken here to be elliptical.

A. Kounine, M. Mannelli, M. Martinez, K. Mönig, G. Myatt, A. Olshevsky, A. Passeri, Ch. Paus, M. Pepe-Altarelli, P. Perret, B. Pietrzyk, D. Reid, M. Roney, D. Schaile, D. Schlatter, T.J. Smith, R. Tenchini, F. Teubert, P. Wells.

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